Round Waveguide with Double Lining

By HANS-GEORG UNGER

(Manuscript received September 22, 1959)

Doubly lining the walls of round waveguide with a base layer of dissipative material and a top layer of low loss material provides mode filtering for TE_{01} transmission and reduces TE_{01} loss in bends. The effects of thin layers are calculated as perturbations of the empty waveguide characteristics. For best performance, the dissipative layer should have low permittivity and high loss factor. The layers should be only a few mils thick.

I. INTRODUCTION

Transmission of the TE_{01} wave in round waveguide is degraded by manufacturing and laying imperfections.[1] To reduce the effects of mode conversion at manufacturing imperfections, mode filters are required. To reduce the effects of laying curvature, the phase constant of the TM_{11} wave must be made different from the TE_{01} phase constant.

Instead of using mode filters, lining the waveguide wall with a thin layer of dissipative material has been suggested.[2,3] Lining the waveguide with a low-loss material reduces the bending effects.[4]

A waveguide with a double lining was proposed by S. E. Miller to solve both problems. A base layer of dissipative material mainly introduces mode filtering, and a top layer of low-loss material changes the TM_{11} phase.

II. PROPAGATION CHARACTERISTICS

For thin dielectric layers, which fill only a small part of the total cross section, the normal modes may be considered perturbed normal modes of the empty waveguide with ideally conducting walls.

A cavity resonating in a mode with field vectors $\mathbf{E}_0$ and $\mathbf{H}_0$ at frequency ω will change its resonance when a small body V_1 of relative permittivity ϵ is introduced:[5]

$$-\frac{\Delta\omega}{\omega} = \frac{\epsilon_0 \int_{V_1} (\epsilon - 1)\mathbf{E}_1\mathbf{E}_0^* \, dV}{\epsilon_0 \int_V \mathbf{E}_0\mathbf{E}_0^* \, dV + \mu_0 \int_V \mathbf{H}_0\mathbf{H}_0^* \, dV}, \tag{1}$$

where ϵ_0 and μ_0 are the permittivity and permeability of the unperturbed cavity, $\mathbf{E}_1$ is the resulting field vector within the volume V_1 and V is the total volume of the cavity. The asterisk denotes a conjugate complex value.

If the cavity is a section of a cylindrical waveguide and if V_1 is also cylindrical, so that ϵ is independent of the axial distance, then the change in resonance frequency (1) of the cavity may be related to a change in the propagation constant γ of the waveguide:

$$\frac{\Delta\gamma}{\gamma} = -\frac{v}{u}\frac{\Delta\omega}{\omega}, \tag{2}$$

where v and u are the phase and group velocities of the unperturbed waveguide.

Although the internal field $\mathbf{E}_1$ is unknown, it is often possible to determine it from elementary boundary conditions. For example, when $\mathbf{E}_0$ is perpendicular to the boundary of V_1,

$$\mathbf{E}_1 = \frac{1}{\epsilon}\mathbf{E}_0 \tag{3}$$

or when $\mathbf{E}_0$ is parallel to the boundary of V_1,

$$\mathbf{E}_1 = \mathbf{E}_0 . \tag{4}$$

For the circular waveguide with a thin dielectric layer of permittivity $\epsilon(r)$ adjacent to the wall, either (3) or (4) will determine $\mathbf{E}_1$ from $\mathbf{E}_0$. Substituting the normal mode fields of circular waveguide for $\mathbf{E}_0$ and $\mathbf{H}_0$, these expressions for the change in propagation constant are obtained:

$$\text{TM}_{nm} \text{ waves:} \quad \frac{\Delta\gamma}{\gamma_{nm}} = \frac{1}{a}\int_0^a \frac{\epsilon - 1}{\epsilon}\,dr, \tag{5}$$

$$\text{TE}_{nm} \text{ waves:} \quad \frac{\Delta\gamma}{\gamma_{nm}} = \frac{1}{1 - \nu_{nm}^2}\frac{n^2}{k_{nm}^2 - n^2}\frac{1}{a}\int_0^a \frac{\epsilon - 1}{\epsilon}\,dr, \tag{6}$$

$n \neq 0$

$$\text{TE}_{0m} \text{ waves:} \quad \frac{\Delta\gamma}{\gamma_{0m}} = \frac{1}{1 - \nu_{0m}^2}\frac{k_{0m}^2}{a^3}\int_0^a (\epsilon - 1)(a - r)^2\,dr, \tag{7}$$

where a is the radius of the waveguide, k_{nm} is the mth root of $J_n(x) = 0$ for TM_{nm} waves and the mth root of $J_n'(x) = 0$ for TE_{nm} waves and $\nu_{nm} = \omega_{cnm}/\omega$ with cutoff frequency ω_{cnm}. Complex permittivities will cause a complex $\Delta\gamma$ corresponding to a change in phase and attenuation constant.

Equations (5), (6) and (7) hold for waveguides with ideally conducting walls. For walls of finite conductivity the modes of the plain pipe suffer wall current losses. These losses are changed by the presence of a dielectric layer. This change, however, is of higher order in the thickness of the layer and can be neglected against the attentuation change caused by losses in the lining from (5) and (6), for most of the modes. For circular electric waves, however, the loss contribution from (7) is of higher order in layer thickness. Then, the change in wall current loss has to be taken into account.

Wall currents of circular electric waves are given by the axial magnetic field at the wall, H_z. The wall current losses are proportional to the square of the wall current amplitudes. Therefore the change in wall current losses $\Delta\alpha$ from its unperturbed value α_0 is:

$$\frac{\Delta\alpha}{\alpha_0} = 2R_e\left(\frac{\Delta H_z}{H_{z0}}\right). \tag{8}$$

From Maxwell's equations for circular electric waves:

$$\frac{\partial H_z}{\partial r} = -j\omega\epsilon\, \epsilon_0 \mathbf{E}_\varphi . \tag{9}$$

Equation (9) can be integrated over a thin dielectric layer adjacent to the walls by using the circumferential electric field $\mathbf{E}_{\varphi 0}$ of the unperturbed mode:

$$\frac{\Delta H_z}{H_{z0}} = \omega^2\mu_0\epsilon_0 \int_0^a (\epsilon - 1)(a - r)\, dr. \tag{10}$$

Then, from (8) and (10)

$$\frac{\Delta\alpha}{\alpha_0} = 2\omega^2\mu_0\epsilon_0 \int_0^a (\epsilon' - 1)(a - r)\cdot dr, \tag{11}$$

with ϵ' from $\epsilon = \epsilon' - j\epsilon''$.

For a double lining, according to Fig. 1, the following expressions are obtained for the change in phase constant:

$$\mathrm{TM}_{nm}: \quad \frac{\Delta\beta}{\beta_{nm}} = \left(1 - \frac{\epsilon_1'}{|\epsilon_1|^2}\right)\delta_1 + \left(1 - \frac{\epsilon_2'}{|\epsilon_2|^2}\right)\delta_2 , \tag{12}$$

$$\begin{aligned} \mathrm{TE}_{nm}: \quad & \frac{\Delta\beta}{\beta_{nm}} = \frac{1}{1 - \nu_{nm}^2}\,\frac{n^2}{k_{nm}^2 - n^2}\left[\left(1 - \frac{\epsilon_1'}{|\epsilon_1|^2}\right)\delta_1\right. \\ n \neq 0 \quad & \qquad\qquad \left. + \left(1 - \frac{\epsilon_2'}{|\epsilon_2|^2}\right)\delta_2\right], \end{aligned} \tag{13}$$

$$\mathrm{TE}_{0m}: \quad \frac{\Delta\beta}{\beta_{0m}} = \frac{k_{0m}^{\ 2}}{3(1 - \nu_{0m}^2)}\left[(\epsilon_1' - 1)\delta_1^3 + (\epsilon_2' - 1)(\delta^3 - \delta_1^3)\right]; \tag{14}$$

and for the change in attenuation constant:

$$\mathrm{TM}_{nm}: \qquad \frac{\Delta\alpha}{\beta_{nm}} = \frac{\epsilon_1''}{|\epsilon_1|^2}\delta_1 + \frac{\epsilon_2''}{|\epsilon_2|^2}\delta_2, \tag{15}$$

$$\mathrm{TE}_{nm}: \qquad \frac{\Delta\alpha}{\beta_{nm}} = \frac{1}{1-\nu_{nm}^2}\frac{n^2}{k_{nm}^2 - n^2}\left(\frac{\epsilon_1''}{|\epsilon_1|^2}\delta_1 + \frac{\epsilon_2''}{|\epsilon_2|^2}\delta_2\right), \tag{16}$$

$n \neq 0$

$$\begin{aligned}\mathrm{TE}_{0m}: \qquad \frac{\Delta\alpha}{\beta_{0m}} = &\frac{k_{0m}^2}{3(1-\nu_{0m}^2)}[\epsilon_1''\delta_1^3 + \epsilon_2''(\delta^3 - \delta_1^3)] \\ &+ \frac{\alpha_{0m}}{\beta_{0m}}\frac{k_{0m}^2}{\nu_{0m}^2}[(\epsilon_1' - 1)\delta_1^2 + (\epsilon_2' - 1)(\delta^2 - \delta_1^2)],\end{aligned} \tag{17}$$

where $\delta = d/a$ is the relative thickness of a lining. Index 1 refers to the base layer and index 2 to the top layer.

Note that the change in propagation constants is of first order in δ for all TM_{nm} waves and for the TE_{nm} waves with $n \neq 0$. It is of third order only for circular electric waves. The second-order term in the expression for added TE_{0m} loss, representing increase in wall current loss, is only of significance for a single low-loss lining.

III. DESIGN OF DOUBLE LINING

It can easily be seen that the present placement of dissipative and low loss material is the best one. The present design objectives are: the change in TM_{11} phase should be as large as possible and the undesired mode loss should be as much as possible, while the added TE_{01} loss should remain small. For unwanted modes, it does not matter where the dissipative material is placed in the lining, since the integrand in (5) and (6)

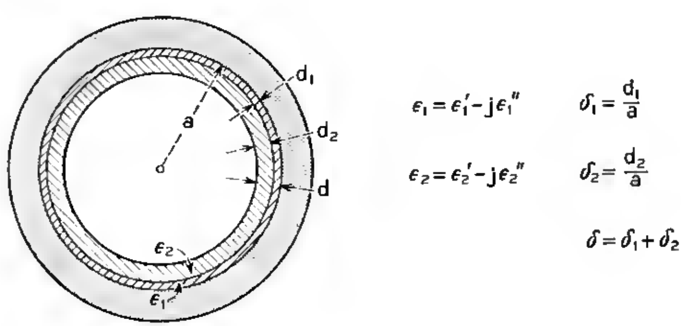

Fig. 1 — Round waveguide with double lining.

depends only on the permittivity. A dissipative material of certain thickness will cause the same perturbation no matter what distance from the wall it has. The relation is quite different for circular electric waves, since in (7) the integrand contains the square of the distance. In order to keep the perturbation small the dissipative material should be placed as close to the wall as possible, so that the dissipative material is being moved into the region of smallest electric field of the circular electric wave.

The same rules apply for dielectric constant and loss factor of the lossy material, as had previously been found for the single lossy lining.[3] The dielectric constant should be low and the loss factor high.

The selection of a suitable double lining is best demonstrated by a numerical example: $\epsilon_1' = 3$ and $\epsilon_1'' = 1.5$ is as close as present materials can be hoped to approach the above design rules for the dissipative layer;[3] $\epsilon_2' = 2.5$ and $\epsilon_2'' = 2.5 \times 10^{-3}$ are electrical properties of common low-loss materials. With $\Delta\alpha/\alpha_{01} = 0.2$, the added TE_{01} loss in the lined pipe is limited to 20 per cent of the TE_{01} loss in the plain pipe. Equation (17) is now reduced to a relation between δ_1 and δ_2. For a 2 inch I.D. copper pipe operated at 55.5 kmc, this relation is plotted in Fig. 2. On the left-hand border a dissipative layer of $\delta_1 = 0.00136$ alone adds 20 per cent to the TE_{01} loss. On the right-hand side it is a single low loss layer of $\delta_2 = 0.0090$.

To decide on a suitable combination of δ_1 and δ_2 two characteristic

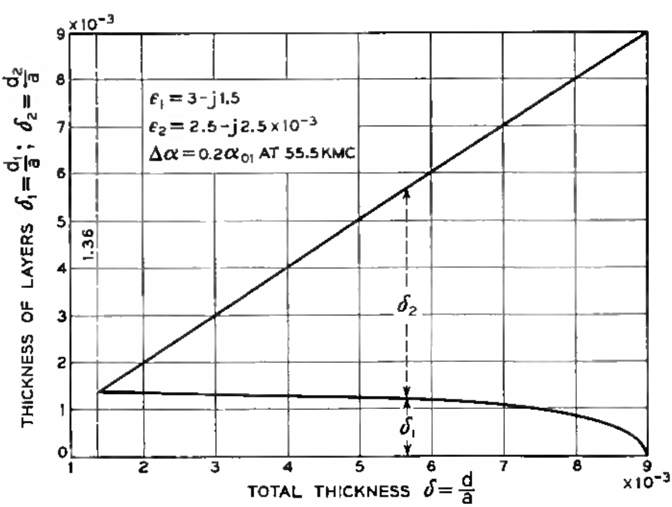

Fig. 2 — Double lining of 2-inch I.D. copper pipe.

quantities of the lined waveguide have been plotted over the same abscissa (Fig. 3). First, there is the added TE_{12} loss according to (16). The TE_{12} mode is most seriously coupled to TE_{01} at all kinds of manufacturing imperfections. To reduce the degrading effects on TE_{01} transmission of such imperfections it is most important to absorb TE_{12} power. Added TE_{12} loss is therefore a good measure for the mode filtering ability of the waveguide.

Secondly, a radius of curvature of a continuous bend is plotted that adds another 20 per cent of α_{01} to the TE_{01} loss. Such a radius characterizes laying imperfections that might be tolerated without exeessive TE_{01} loss. The smaller this radius, the more freedom in laying is allowed.

The added TE_{12} loss is highest for a single lining of dissipative material, on the left-hand border. But when a low-loss layer is added, $\Delta\alpha$ of

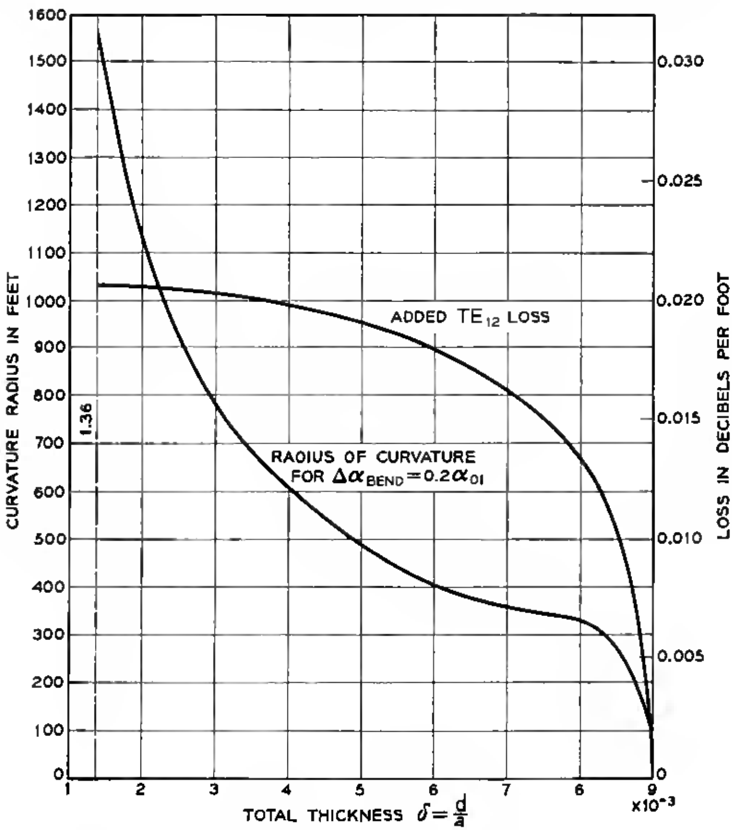

Fig. 3 — Mode filtering and bending in copper pipe with double lining of Fig. 2.

TE_{12} does not change very much at first. Even at $\delta = 0.005$, it decreases only by 9 per cent. On the other hand, the tolerable radius of curvature decreases quite rapidly from its high value for a single dissipative layer. At $\delta = 0.005$, R falls to less than one-third of its highest value. The curve is more level, however, for a substantial thickness of the second, low loss layer.

A good choice for a waveguide that has efficient unwanted mode absorption and allows much freedom in bending is $\delta = 0.005$ or, from Fig. 2,

$$\frac{\delta_2}{\delta_1} = 3.$$

The low-loss layer should be three times heavier than the dissipative layer.

IV. CONCLUSIONS

Applying two dielectric layers to the internal waveguide surface, a base layer of dissipative material and a top layer of low-loss material, is a useful modification for circular electric wave transmission. The lining changes attenuation and phase constant of circular electric waves very little. On the other hand, the dissipative layer effectively increases unwanted mode loss and the low-loss layer shifts their phase constant, in particular that of TM_{11}.

Thus, in combining the characteristics of a dissipative lining and a low-loss lining, this structure provides mode filtering and reduces TM_{11} conversion in bends. The mode filtering characteristics are nearly equivalent to those of a pipe with a single lossy lining. The additional low-loss lining gives much more freedom in bending.

REFERENCES

1. Warters, W. D. and Rowe, H. E., Transmission Deviations in Waveguide Due to Mode Conversion: Theory and Experiment, Conv. on Long-Distance Transmission by Waveguide, London, 1959; Proc. I.E.E., Suppl. 13.
2. Malin, V. V., The Influence of a Semiconductive Film on the Attenuation in a Waveguide of Circular Cross Section, Rad. Tech. i Elek., **1,** January 1956, p. 34.
3. Unger, H., Round Waveguide with Lossy Lining, Proc. of Symposium on Millimeter Waves, New York, 1959.
4. Unger, H., Circular Electric Wave Transmission in a Dielectric-Coated Waveguide, B.S.T.J., **36,** September 1957, p. 1253.
5. Müller, J., Untersuchungen über elektromagnetische Hohlräume. Hochfreq. Technik, **54,** 1939, p. 157.